Physics 201 Name KEY

Exam 1 – Electrostatics

February 11, 2009

This is a closed book examination but during the exam you may refer to a 3"x5" note card with words of wisdom you have written on it. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on all questions.

1. [4 PTS] By comparison with the force of gravity, the electrostatic force between an electron and a proton
   a) is repulsive and very much weaker.
   b) is repulsive and very much stronger.
   c) is just about the same magnitude.
   d) is also attractive but very much weaker.
   (e) is also attractive but very much stronger.
   f) is not measured in Newtons and hence cannot be compared.

   (+) (−) Attractive

   $F_q = \frac{q_1 q_2 k_e}{r^2} \gg \frac{G m_1 m_2}{r^2} = F_g$

2. [4 PTS] Two charges, Q_1 and Q_2, are located in an empty region of space. The force on Q_1 is 12 N when Q_2 is 2 meters away. What is the force on Q_2 when Q_1 is moved 2 meters further away so that 4 meters now separate the charges?
   (a) 3 N
   b) 6 N
   c) 12 N
   d) 24 N
   e) 48 N

   $F = \frac{k q_1 q_2}{r^2}$ $r_2 = 2r_1$ so $F_2 = \frac{k q_1 q_2}{(2r_1)^2} = \frac{1}{4} F_1$

   $F_2 = 3N$

3. [4 PTS] A volume of space is found to have a constant potential of 12 volts everywhere within it. It follows that in that region
   a) the electric field is also constant and negative.
   (b) the electric field is zero.
   c) the potential gradient is a nonzero positive constant.
   d) none of these.

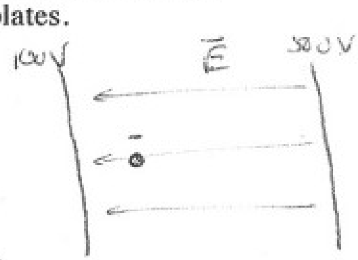
   $E = -\frac{dV}{dr}$

   so if V = constant then $\frac{dV}{dr} = 0$

4. [4 PTS] An electron is placed between two parallel plates. One plate is held at 100 V and the other is held at 500 V. The electron is placed closer to the 100 V plate. Which way does the electron move?
   a) The electron does not move.
   b) The electron moves toward the 100 V plate.
   (c) The electron moves toward the 500 V plate.
   d) The electron moves parallel to the plate and out the nearest side.
   e) The electron moves in a circle parallel to both plates.

   $\vec{F} = q\vec{E}$

   Electron moves toward highest potential (vs field)



5. [4 PTS] A hollow metal (conducting) box is placed 8.6 cm away from a balloon with 2.6 C of charge. The equal potential lines (voltage) from the charge are proportional to 1/r. Which statement is true about the electric field and/or voltage inside the box?
   a) The electric field and voltage are both zero.
   b) The voltage is zero.
   c) The voltage is larger on the side of the box closer to the charge.
   d) The electric field is larger on the side of the box closer to the charge.
   e) The electric field is zero.

The electric field is zero inside a conductor
The voltage is constant,

6. [4 PTS] Two nested spheres surround a negative charge. The larger sphere has a radius that is 3 times larger than the inner sphere. The electric flux through the larger sphere
   a) is 9 times less than the flux through the smaller sphere.
   b) is 3 times less than the flux through the smaller sphere.
   c) is the same as the flux through the smaller sphere.
   d) is zero.
   e) is 3 times more than the flux through the smaller sphere.
   f) is 9 times more than the flux through the smaller sphere.
   g) can not be determined.

$\Phi_E = \frac{Q_{ENC}}{\epsilon_0}$

Q_{ENC} is the same so the flux is the same

Please do the next two problems on additional paper.

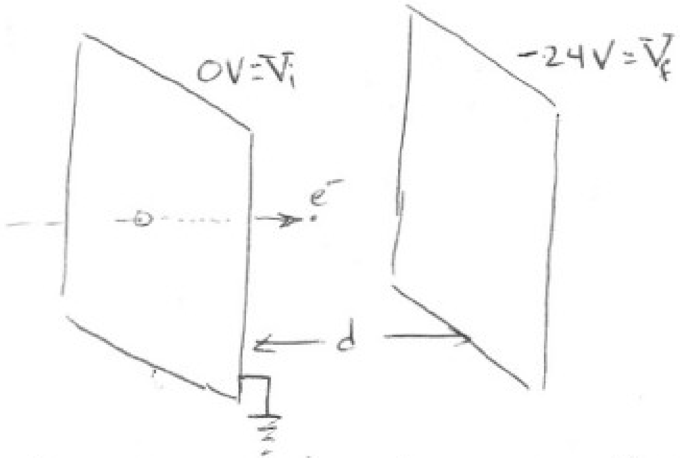
7. [10 PTS] An electron is shot through a small hole in a large plate at 1% the speed of light ($v_e = 3\times10^6 \, m/s$) towards a second plate. Both plates are parallel to each other and are separated by 12 mm. The plate with the small hole in it is grounded while the second plate is connected to a battery at -24 volts. The mass of an electron is $m_e = 9.1\times10^{-31} kg$ and the charge on an electron is $q_e = 1.6\times10^{-19} C$.
   a) What is the velocity of the electron when it reaches the second plate?
   b) How long does it take the electron to travel the 12 mm? NOTE: Think kinematics.

8. [10 PTS] A ball of negative charge has a charge density, $\rho(r) = -3.0\times10^{-6}$ r C/m^4, which increases with radius. The ball has a radius $R_B = 0.015$ m.
   a) What is the total charge in the ball?
   b) What is the electric field inside and outside the sphere?
   c) What is the potential difference between $r_1 = .004$ m and $r_2 = .010$ m?

Useful trigonometric relationships: $\sin^2(\theta) + \cos^2(\theta) = 1$
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ and $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

Exam 1 2009

(7)



$\vec{v}_i = 3\times10^6\ m/s$ $\quad v_f = ?$

$m_e = 9.1\times10^{-31}\ kg$

$q_e = -1.6\times10^{-19}\ C$

$\Delta\mathbb{V} = 24V$

$d = 0.012\ m$

(a) Use conservation of energy $E_i = E_f$

$$\frac{1}{2}mv_i^2 + q\mathbb{V}_i = \frac{1}{2}mv_f^2 + q\mathbb{V}_f$$

$$\frac{1}{2}mv_i^2 + q(\mathbb{V}_i - \mathbb{V}_f) = \frac{1}{2}mv_f^2$$

$$v_i^2 + \frac{2q}{m}(\mathbb{V}_i - \mathbb{V}_f) = v_f^2$$

$$\left[(3\times10^6\ \tfrac{m}{s})^2 - \frac{2(1.6\times10^{-19}C)}{(9.1\times10^{-31}kg)}(0+24)\right]^{1/2} = v_f = \boxed{7.5\times10^5\ m/s}$$

- ☑ units
- ☑ $v_f < v_i$ as expected
- ☑ as $\mathbb{V}_f \downarrow$ $v_f \downarrow$
- ☑ as $q \uparrow$ $v_f \downarrow$

(b) Use kinematics

$d = v_{AVG}\cdot t$ so $t = \dfrac{d}{(v_f+v_i)/2}$

or

$v_f^2 - v_i^2 = 2ad \qquad v_f = v_i + at$

$a = \dfrac{(v_f^2 - v_i^2)}{2d} \qquad \dfrac{v_f - v_i}{a} = t$

$= \dfrac{(v_f - v_i)(v_f + v_i)}{2d} \qquad \dfrac{2d}{(v_f+v_i)} = t$

$t = \dfrac{2\cdot(0.012)}{(3\times10^6 + 7.5\times10^5)} = 6.4\times10^{-9}\ s = \boxed{6.4\ ns}$

☑ units

Without $\vec{E}$ the e^- would only take 4.0 ns

(8)

$\rho(r) = -3.0\times10^{-6}\, r \ \frac{C}{m^4}$ $\epsilon_0 = 8.84\times10^{-12}\ \frac{C^2}{Nm^2}$

$R_B = 0.015\,m$

(a) $Q_{TOT} = \int_0^{R_B} \rho(r)\, 4\pi r^2\, dr$ Add up the shells of thickness dr

$= -3.0\times10^{-6}\cdot 4\pi \int_0^{R_B} r^3 dr = -3.0\times10^{-6}\, 4\pi \left.\frac{r^4}{4}\right|_0^{R_B} = -3.0\times10^{-6}\pi R_B^4$

$\boxed{Q_{TOT} = -4.8\times10^{-13}\, C}$ [3×10^{6} ions]

(b) Use Gauss' Law

$\oint \vec{E}\cdot\hat{n}\, dA = \frac{Q_{ENC}}{\epsilon_0}$

a spherical symmetry ~

$dV = 4\pi r^2 dr$

$\oint dA = 4\pi r^2$

INSIDE: $E\, 4\pi r^2 = \frac{\int \rho(r)\, dV}{\epsilon_0} = \frac{1}{\epsilon_0}\int -3.0\times10^{-6}\, 4\pi r^3 dr$

$E\, 4\pi r^2 = \frac{-3.0\times10^{-6}\pi r^4}{\epsilon_0}$

$\vec{E} = \frac{-3.0\times10^{-6}}{4\epsilon_0}\, r^2\, \hat{r}$

OUTSIDE: $E\, 4\pi r^2 = \frac{Q_{TOT}}{\epsilon_0}$

$\vec{E} = \frac{Q_{TOT}}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$

$= \frac{-3.0\times10^{-6} R_B^4}{4\epsilon_0}\frac{1}{r^2}\hat{r}$

(c) $r_1 < r_2 < R_B$ inside sphere

$V = -\int \vec{E}\cdot d\vec{r} = -\frac{-3.0\times10^{-6}}{4\epsilon_0}\int_{r_1}^{r_2} r^2 dr$

$= \frac{3.0\times10^{-6}}{4\epsilon_0}\left.\frac{r^3}{3}\right|_{r_1}^{r_2} = \frac{3.0\times10^{-6}}{12\epsilon_0}\left(r_2^3 - r_1^3\right) = \Delta V$

$r_1 = 0.004\, m$

$r_2 = 0.010\, m$

$\boxed{\Delta V = 0.026\, V}$